Full waveform microseismic inversion using differential evolution algorithm

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Outline

1. Introduction
2. Problem formation
3. Propose method
4. Synthetic Simulation
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Surface monitoring during hydraulic fracturing

- **Geophone Array**
- **Microseismic Events**
- **Hydraulic Fracture**
- **Well is turned horizontal**

*Geological Structures*
Summary

- Low oil price urges for cost-effective **long-term** monitoring
- Increasing interests on surface geophone array monitoring
  - *Low cost* comparing to wellbore array
  - Good azimuth angle coverage
  - *Long term* monitoring
- Microseismic events is a good indicator of subsurface structure changes
  - *Event location*
  - *Source mechanism*
- Processing pipeline
  - Pre-processing (QC and de-noise)
  - Event detection
  - *Event localization*
  - Finding *source mechanism*
Previous work on event localization

- Digitize the entire monitoring space into small blocks (grid nodes)
- Semblance (hours to days)
  [Gharti et al., 2010, Frantiek* et al., 2014]
  - Search all possible grid nodes using simple but fast method.
  - Rely on the coherent signal energy across the receiver array.
  - Low computation requirement, but might give misleading or imprecise results.
- Back-propagation (days to months)
  [Gajewski and Tessmer, 2005, Haldorsen et al., 2012]
  - Reverse time and back propagate wave field in digitized grids based on wave equations.
  - Take advantage of full waveform information.
  - Effective but expensive (time and memory), especially for 3D elastic wave.
  - Sensitive to model error, can have poor focusing.
- Both methods were developed using single component data
Example of traditional methods

(a)

(b)

Figure 1: Event localization from (a) semblance based method and (b) reverse-time based method.
3-component data and source mechanism

- 3-component (3-C) data is becoming popular
- Source mechanism is also important in reservoir monitoring
- Identify the source mechanism along with the localization becomes possible

(a) 3-C data

(b) Moment tensor
Problem formation

- **Assumptions**
  - Event origin time is given by event detection
  - Source waveform is available through wavelet estimation
  - Only **AWGN** is considered after pre-processing
  - **Isotropic** lossless layered velocity model

- **Forward modelling of 3-C data**
  - For complicated model, Finite-difference is used to compute Green’s function
  - For layered velocity model, Green’s function for *p-wave and s-wave* can be obtained by Generalized Ray Theory [Ben-Menahem and Singh, 2012]
  - Separate moment tensor and wave propagation due to **isotropy** of the media
Physical model

For $i^{th}$ source and receiver pair, a Green’s function $g[i]$ satisfies

$$u[i] = (g[i] * w)m$$

where $u[i]$ is the data received, $w$ is the source wavelet and $m$ is the moment tensor.

Denote the convolution by $G[i] \triangleq g[i] * w$. Stack $G[i]$ into a big matrix $G$ and data matrix $u[i]$ into $u$, we have

$$u = Gm$$  \hspace{1cm} (1)$$

where both $G$ and $m$ are unknown.

For a set of receiver locations, fixed velocity model and source wavelet, $G$ is only \textbf{a function of source location $s$}, thus

$$u = G(s)m$$  \hspace{1cm} (2)$$
Minimization problem

■ Original problem

\[
\text{Minimize } \| u - G(s)m \| \quad (3)
\]

■ For a fixed \( s \), \( m \) can be estimated by least squares

\[
\hat{m}(s) = (G^H(s)G(s))^{-1}G^H(s)u \quad (4)
\]

■ New problem

\[
\text{Minimize } J(s) \triangleq \| u - G(s)\hat{m}(s) \| \quad (5)
\]

■ In most cases, \( J(s) \) is a highly non-linear, non-convex function of \( s \).
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Search for the minimum

- **Grid search**
  - Small model, coarse grid
  - Green’s function of every source-receiver pair is evaluated
  - Minimum is guaranteed

- **Differential Evolution algorithm (DE)**
  - A smart way to sample the parameter space by population
  - Mutation is introduced for each generation (iteration) based on the current population
  - Selected mutants are compared with current population, the better one goes into the next generation
  - Requires fewer evaluations of forward modelling (computation of Green’s function)
Differential evolution

- Initialization: randomly select an initial population of $D$ agents consisting a set of parameters
- Mutation $v_p$:
  \[ v_p = x_{p1} + F(x_{p2} - x_3) \]  
  where $F \in [0, 2]$, $x_{p1}$ to $x_{p3}$ are distinct and randomly selected from current population.
- Crossover:
  \[ u_j = \begin{cases} 
  v_j & \text{if } p_j \leq C \text{ or } j = RI \\
  x_j & \text{otherwise}
  \end{cases} \]  
  where $p_j \sim U(0, 1)$, $C \in [0, 1]$, and random index($RI$) is among \{1, \cdots, $D$\}.
- Selection: Choose between $u_i$ and $x_i$ and keep the one with lower cost function $J(s)$. 
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Figure 2: Simulation setup: (i) array geometry and (ii) sample data with 25 dB PSNR: (a)x, (b)y, (c)z components.
Simulation setup

- **15 × 15** surface geophone array, **double-couple** moment sensor shown below:

\[
MT = \begin{bmatrix}
0.4330 & -0.2500 & 0.7500 \\
-0.2500 & -0.4330 & 0.4330 \\
0.7500 & 0.4330 & 0.0000
\end{bmatrix}
\]  

(8)

- Use **PSNR** as the measurement of noise level:

\[
\text{PSNR} = 20 \log_{10} \frac{D_{\text{max}}}{\sigma}
\]

(9)

where \(D_{\text{max}}\) is the maximum magnitude of a trace and \(\sigma\) is the standard deviation of AWGN.

- The model size is of **30 × 30 × 15** grid points with **40m** spatial resolution
Details about DE algorithm

- Off-grid point
  - Move to the **nearest grid node**
  - Green’s function of each node will only be **evaluated once** in the simulation

- Population size
  - Rule of thumb: population size is *5 to 10 times* the dimension of parameter space
  - In our example, the dimension of parameter space is three (x, y, z): **population size is 30**

- Accuracy measurement
  - The spatial resolution is 40m, the half diagonal distance is about 30m \((20\sqrt{2})\)
  - 60m error will be acceptable, **30m** error will be a good estimation

- Termination condition
  - DE program can be restarted at any iteration as long as the population is saved
  - Gradually increase the number of iteration until the cost function is stable
Convergence rate by iteration

- **Mean (meters)**
  - 0
  - 200
  - 400
  - 30

- **Standard Deviation (meters)**
  - 0
  - 200
  - 400

- **Mean of mismatch**
- **Cost function**
- **30 m error reference**
- **Std of mismatch**
Population convergence as iteration increases

- Population converges slower than the estimated error
- Dot color and size indicate number of iterations
Simulation results

- **Accuracy**
  - acceptable accuracy (60m error) within 40 iteration
  - good accuracy (30m error) within 60 iteration

- **Robustness**
  - Reach good accuracy in 100 iteration down to 0 dB PSNR
  - Event detection will break before the localization algorithm

- **Computation requirement**
  - Grid search: $30 \times 30 \times 15 \times 225 = 3,037,500$ evaluation of Green’s function (4 days)
  - DE algorithm ($C = 0.5$): $15 + 0.5 \times 30 \times 60 \times 225 = 205,875$ evaluation of Green’s function (6 hours)
  - DE evaluates only 6.7% of all Green’s functions
The proposed method integrates **moment tensor inversion** and **event localization**

- **Reduce the dimension** of parameter space from 9 to 3 using proposed scheme
- Synthetic simulation illustrates a **good accuracy** of proposed method within reasonable time
- Differential evolution method **evaluates significantly fewer Green’s functions** than grid search method
References


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